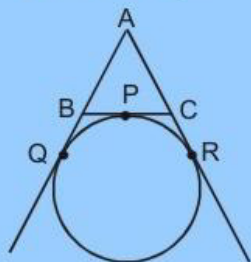# Applications

A circle touches the side BC of a ΔABC at P and touches AB and AC when produced at Q and R respectively then

$$AQ = \frac{1}{2}(\text{Perimeter of } \Delta ABC).$$

$AQ = AR$ ....(i)
$BQ = BP$ ....(ii)
$CP = CR$ ....(iii)
[Tangents drawn from an external point to a circle are equal]
Now, perimeter of ΔABC
$= AB + BC + CA$
$= AB + BP + PC + CA$
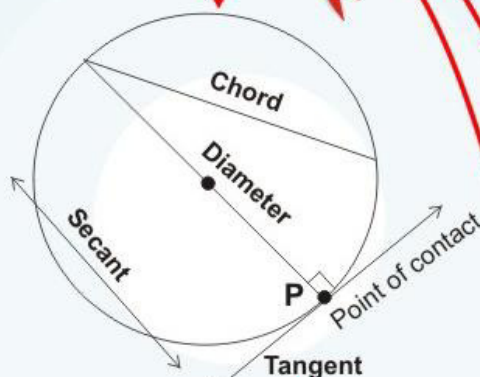$= (AB + BQ) + (CR + CA)$ [From (ii) and (iii)]
$= AQ + AR = AQ + AQ$ [From (i)]
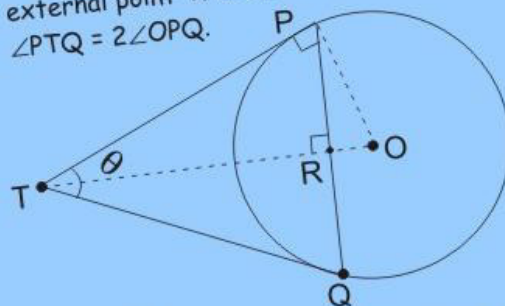$$AQ = \frac{1}{2}(\text{Perimeter of } \Delta ABC).$$

Special Note :
1. If quadrilateral ABCD is circumscribing a circle, then $AB + CD = AD + BC$.
2. If all the sides of a parallelogram touches a circle, then the parallelogram is a rhombus.

CIRCLES

Chord
Diameter
Secant
Point of contact
P
Tangent

Special case of secant with only one point of contact.

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ.$
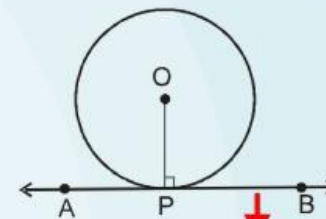

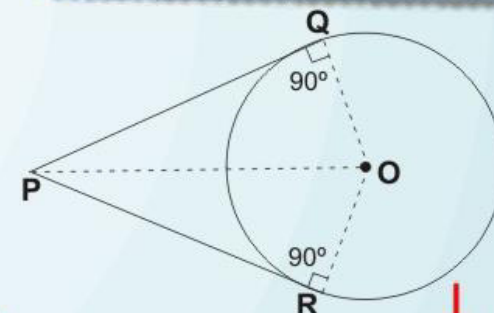In ΔPTQ
$TP = TQ$, hence $\angle TQP = \angle QPT = 1/2 (180 - \theta)$

Since $\angle OPT = 90°$, $\angle OPQ = \angle OPT - \angle TPQ$

$$\angle OPQ = 90° - \frac{1}{2}(180° - \theta)$$

$$\Rightarrow \angle PTQ = 2\angle OPQ.$$

A tangent to a circle is perpendicular to the radius through the point of contact.
i.e. $OP \perp AB$

Length of two tangent drawn from external point are equal
In ΔOQP & ΔORP
$\angle OQP = \angle ORP$ (Each 90°)
$OP = OP$, (Common)
$OQ = OR$ (radius)
$\therefore \Delta OQP \cong \Delta ORP \Rightarrow PQ = PR$

Results :
(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle POQ = \angle POR$
(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle OPQ = \angle OPR$

NCERT / X / Circles